**About the True Equilibrium in the Population of the Plants**

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**ABSTRACT**

On the basis Mendel's experiments, a mathematical model is constructed that describes the results of these experiments in a wide range of parameters. This model is compared with the Hardy -Weinberg logistic model, based only on probabilistic ideas about the presence of dominant and recessive alleles in the chromosomes of living organisms. There is shown that in the mathematical model of Mendel's experiments, based on real patterns of plant development, there are equilibrium positions between the dominant and recessive forms. Its consists in the fact that with an increase in the number of generations of the number all of dominant and recessive phenotypes of organisms, with any number of sings, they quickly equalize and then synchronously (in the absence of death of organisms) increase together, seeking asymptotically to a stable isolated equilibrium position of the type of a multidimensional node. This newly discovered behavior of the dominant and recessive forms in the vicinity of the equilibrium position (true) differs significantly from the logistic equilibrium position in the Hardy -Weinberg principle, built without taking into account the real patterns in the plant population.

**Key words:** Plant; Chromosome; Crossing; Hybrid; Allele

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**INTRODUCTION**

In 1865, Mendel made a presentation at the Society of Naturalists in Brynn about experiments on plant hybrids[1]. This presentation gave birth to the development of genetics as a science, although not immediately the content of the speech became of known to the scientific community and was appreciated[2]. More than 30 years after this speech, when the results works of Mendel were reopened and confirmed experimentally in the works of Correns[3], and de Vries[4].

Over the past 150 years after the speech of Mendel, a chromosome theory of heredity was created, which gave a molecular explanation to the results of the experiments of Mendel[5-12]. The same time after of the second discovery of Mendel's experiments, there appeared works in which it was noted that in Mendel's experiments there was a steady increase in the number of dominant alleles in populations, which indicated the absence of equilibrium positions in plant populations that obeyed Mendel's law.

In this regard, Yule[13] purely mathematically proved that in the case of free crossing in the population of heterozygous forms, there is an equilibrium between the number of dominant and recessive forms. Continued these studies Hardy[14], who derived the formula for the distribution of genotypes in freely crossbreeding populations. Regardless of him and even earlier, Weinberg[15] established the same formula. This formula was called as principle the Hardy - Weinberg and became widespread. However, for the mathematical derivation of this formula, very strong assumptions are used: lack of choice in organisms, infinity of the population, accidental crossing of the population's organisms with each other, uniform distribution of male and female individuals, absence of mutation and genetic drift. The totality of these assumptions precludes the possibility of realizing such populations in nature. Therefore, this principle cannot be confirmed experimentally. Its only advantage is that it has equilibrium positions (a finite ratio of the numbers of dominant and recessive forms). However, these equilibrium positions are formed for any initial contents of these forms. Consequently, the set of equilibrium positions is a continuous manifold, and therefore they are asymptotically unstable, since any small perturbation can translate the system from one equilibrium position to another[16,17]. The combination of necessary mathematical conditions in the derivation of the Hardy -Weinberg formula does not alleviate this advantage.

**METHODS**

In this paper, a mathematical analysis of the sequences of obtaining the values ​​of the number of phenotypes of organisms in Mendel's experiments was carried out. It is found that the patterns of inheritance of constant - differing sings, experimentally established by Mendel, are described by special algebraic relations, the basis of which are geometric progressions. Mendel and subsequent researchers did not pay attention to this. Naturally, Mendel could not trace the patterns of inheritance on experiments with a large number of generations. However, the mathematical model obtained in this paper, accurately confirmed by Mendel's opaque in the field of conditions of their conduct, allows one to cram into the inheritance processes of sings with a large number of generations. A numerical and qualitative analysis of the equations of the mathematical model with an increase in the number of generations with an arbitrary number of pairs of constant - differing features was carried out.

**RESULTS**

There is shown that in the mathematical model of Mendel's experiments, based on real patterns of plant development, there are equilibrium positions between the dominant and recessive forms. It is shown that with an increase in the number of generations of the number all of dominant and recessive phenotypes of organisms with any number of sings quickly equalize and then synchronously (in the absence of death of organisms) increase together. In this case, the system asymptotically strive to achieve an isolated stable equilibrium position of the node type in a multidimensional space with coordinates in the form of quantities inversely proportional to the number of different phenotypes of organisms.

**DISCUSSION**

Thus, for finding equilibrium positions there is no need to use such stringent conditions which definition of the Hardy – Weinberg principle. Moreover, the regularities obtained by Mendel are based on clear biological processes in cells, in chromosomes. Moreover, the behavior of the system in the vicinity of the equilibrium position of the mathematical model of Mendel's experiments has a clear physical meaning. While one of the authors of the Hardy -Weinberg principle wrote that, a purely mathematical probabilistic description of random crossing is caused by the impossibility of having any physical justification for it[13].

In the experiments of Mendel, the plants were pollinated by self - pollination, since many plants are pollinated by this method[18]. In addition, recently in the neurobiology of plants convincingly shown[19] that even in cross - pollination, for example, bees, insects pollinate plants not in a random way. The so - called "law of place" appears. The bees pollinate the plants of the species from which they started to pollinate in the early morning. This manifests the intellectual ability of plants to manage the surrounding nature for their needs[20,21].

**MONO - HYBRID CROSSING**

Mendel carefully selected plants so that they consistently had differing signs (constant -distinguishing signs) to determine the inheritance patterns of plant sings. Now can to say that, in accordance with the chromosome theory of heredity, two homozygous organisms were crossed, the homologous chromosomes of each of which contain two identical genes. One pair of genes (*AA*) in one organism corresponds to one sing (dominant), and the other pair of genes (*aa*) in another organism corresponds to another sing (recessive). This crossing is called monohybrid. In accordance with Mendel's first law, when crossing homozygous parent forms that differ in one pair of sings, all hybrids of the first generation will be uniform both in genotype (gene composition, *Aa*) and in phenotype (appearance). When the first generation hybrids are crossed in accordance with the second Mendelian law, the predominance of the dominant sing over the recessive one in the ratio 3: 1 for the phenotype and 1: 2: 1 for the genotype will be observed in the offspring. This means that organisms of the first generation  are characterized by a pair of *Aa* genes, and the second generation  is characterized by pairs *AA, Aa, aa* in the ratio 1: 2: 1. Moreover, organisms with *AA* and *Aa* genes have the same phenotype, since the influence of recessive genes affects only when there are two, and not one, in homologous chromosomes.

Suppose that in the generation of  there are *m* organisms and their mean fecundity is *ν*. Then in the generation of  there are *mν* organisms. Taking into account Mendel's second law is necessary to conclude that *mν* is a multiple of four. Then in the generation of  there are  homozygous organisms with *AA* genes, as many homozygous organisms with genes *aa* and  of heterozygous organisms with genes *Aa*



Each of the constituent parts of the second generation gives offspring to the next generation. At the same time, homozygous organisms have homozygous organisms in the progeny, but the total number of corresponding organisms increases by a factor of *ν*. The heterozygous organisms, which have both dominant and recessive genes, also breed homozygous and heterozygous organisms. Taking into account the number of heterozygous organisms in the generation  and their fertility, one have



In sum, homozygous and heterozygous organisms of generation  give the following distribution of organisms in generation 



It is easy to determine that the ratio of the number of organisms of the dominant phenotype to the number of organisms of the recessive phenotype in this generation is 5: 3. In the generation, heterozygous organisms also breed homozygous and heterozygous organisms



Homozygous organisms increase their numbers according to their fertility. In the sum in generation four, the following distribution of progeny is obtained



The ratio of the numbers of the dominant and recessive phenotypes in this generation is 9: 7.

In the *n* generation, arguing in a similar way, one get the following expression for the distribution of organisms



The numbers of the dominant  and recessive  phenotypes in the generation *n* are determined by the following equalities





The numerator on the right - hand side of the last equality is a geometric progression



Therefore, the numbers of the dominant and recessive phenotype are equal, respectively

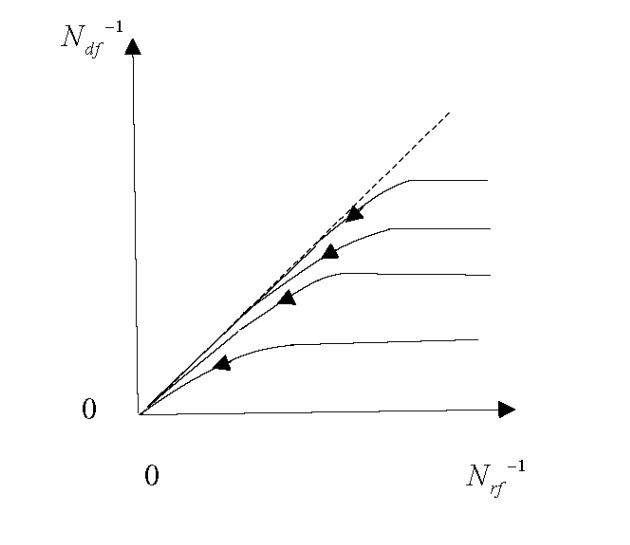
 (1)

The ratio of these numbers is

 (2)

Equation (2) represents a general expression for the ratio of the numbers of the dominant and recessive phenotypes in a mono - hybrid crossing for any number of generation *n* > 1. It depends only on *n*. When *n* = 2, the ratio (2) is 3: 1, which corresponds to Mendel's second law. It follows from (1), (2) that when the number of the generation increases, the numbers of the dominant and recessive phenotypes quite quickly occur (their ratio tends to unity). Absolute values ​​of these numbers grow steadily with increasing number of generation. This is natural, since conditions that are not too remote from the initial state are considered, when the death of organisms is not taken into account due to external influences (aggression of surrounding, climatic and seasonal causes[22,23]). The number of hybrids, which is a component of the number of dominant phenotypes, also grows steadily as the number of the generation increases (*ν* is usually greater than 2). Thus, the assertion that with the passage of time there is a restoration of parental populations and the disappearance of hybrids is erroneous.

Since the numbers of phenotypes grow with increasing *n*, it is convenient to go over to variables, inverses of the numbers of phenotypes, in order to analyze the nature of the desire of the system for the equilibrium position. The set of solutions of the system for various initial values ​​of the number of organisms *m* and the constant value of fecundity *ν* in these variables are represented by trajectories in the phase plane (,) (Figure 1). From which we see that the ratio (2) defines in this plane an intrinsic direction, along which the solutions tend to the equilibrium position at the origin of coordinates of the phase plane. The equilibrium position is isolated, stable and has a node type.

Figure 1. Phase diagram of changes in the numbers of dominant and recessive phenotypes

**BE** – **HYBRID CROSSING**

Let the organisms have two pairs of constant - differing signs. In one pair, the  genes correspond to the dominant sign, and the genes  correspond to the recessive sign. In another pair, the  genes correspond to the dominant sign, and the genes  correspond to the recessive sign. In each pair of signs, Mendel's second law is independent and at the same time acts. This corresponds to the product of expansions

 (3)

since the probability of the product of independent events is equal to the product of the probability of these events. The result of this work are 16 terms

 (4)

Therefore, if in the first generation  there are *m* hybrid organisms, then in the second generation, taking into account their fertility *ν*, there are *mν* organisms and this number must be a gradual number 16. Consequently, the second generation is the sum of the following organisms

 (5)

One distinguish among the 16 terms in (4) those terms that correspond to organisms with the phenotype on the sing of  genes. Such terms are 12



In (4) are terms that correspond to organisms with the phenotype on the sing of genes. This are 4 terms  Thus, the ratio of the number of organisms in the second generation (5) with the  phenotype and with the phenotype  is the same as in the mono - hybrid crossing 3:1. Obviously, the same 3: 1 ratio of the number of organisms with phenotypes of  and of. This manifests the third law of Mendel, discovered by him in experiments with bi - hybrid crossing: the law of independent splitting of sings in the ratio of 3: 1 in the second generation. In this paper, this law follows from a mathematical treatment of bi - hybrid crossing. From the same consideration follows the second part of the third law of Mendel.

Indeed, if one select the terms corresponding to the organisms with the phenotype for the genes from the sum (4), then there are nine such terms. The terms in (4) corresponding to the organisms with the phenotype in the product are three. The same number of terms in (4) for phenotype. These are the terms. One term in (4), corresponding to the organisms with phenotype, this is. Thus, in the second generation (5), the ratio of the number of organisms to the simultaneous presence of some two of the four sings is 9: 3: 3: 1. This is the second part of the third law of Mendel, observed by him in the experiments on bi - hybrid crossing in the second generation.

Mendel did not succeed in elucidating the regularity of bi - hybrid crossing in the following after the second generation. This can be done using a mathematical description, the correctness of which has already been proved. In the transition to the third generation, heterozygous organisms  in (3) are split according to Mendel's second law. If we focus on the ratio of the phenotype counts by the dominant  gene and the recessive gene, then after splitting the heterozygous organism  in the first bracket of the product (3), this product takes the form  where, for brevity, *f* denotes the second bracket in (3). It follows that the ratio of the number of organisms in the phenotypes and  in the third generation is 7: 3.

Similarly, in the fourth generation, the product (3) takes the form, that is, in the fourth generation, the ratio of the numbers of organisms according to the phenotypes and  is 15: 7 and so on. Expanding the sequence of the obtained digits in the relationship, one conclude that in the generation *n* the number of organisms according to the phenotype in the product (3) is a geometric progression



In addition, the number of organisms according to the phenotype  is a geometric progression



Thus, one obtain that the ratio of the number of organisms in the generation *n* according to the phenotypes and  in the bi - hybrid crossing is (6)

It is clear that this formula is also suitable for the ratio of the numbers of organisms according to the phenotypesand in the bi - hybrid crossing in the generation *n*, in view of the equality of rights of brackets in the product (3).

If it is necessary to consider the ratio of the number of organisms to phenotypes with the simultaneous presence of two pairs of sings in the generation after the second, then it is necessary to split both brackets in the product (3) sequentially. Denote the organisms by the phenotype by the symbol. These organisms include organisms with  and genes. Similarly,  denotes the organisms according to the phenotype of,  - organisms according to the phenotype of,  - organisms according to the phenotype of. Then expression (3) in the third generation takes the form



From this it follows that the ratio of the number of organisms according to the phenotypes and, and,and , and in the third generation is. In the fourth generation, the ratio of the number of organisms by the same phenotypes is obtained similarly when the brackets are split in expression (3) .

The construction of these relations continues in the next generation. The numbers participating in the relationships (1, 3, 7, 15,...) are numerators and denominators (6) for the corresponding values ​​of *n*. The indicators and the number of repetitive products in the relationship indicate the number of independent feature pairs (in this case 2). This leads to the following ratios of the number of organisms according to the phenotypes, with the simultaneous presence of two pairs of characters for an arbitrary generation number *n*

 (7)

We introduce the notation of the absolute numbers of organisms in the generation *n* and phenotypes with two dominant genes, with one dominant and one recessive gene and, and with two recessive genes. These numbers, according to (7), are equal



For large values ​​of *n*, these numbers tend to the same value, that is, there is an alignment of the number of organisms with purely dominant genes, purely recessive genes and hybrid genes. Just as with mono - hybrid crossing in the case of bi - hybrid crossings, solutions with an increase in the number of generations can be represented in space with coordinates in the form of inverse numbers. In this case, there are four such coordinates. The equilibrium position of the given system is the origin of the coordinates of the four - dimensional space. This is a special point of the type of a considerate four - dimensional node. The solutions of the system enter into a position of equilibrium in its own direction, defined by the equality of a unit of relation pairwise coordinates of space.

**POLY - HYBRID CROSSING**

If organisms with more than two pairs of characteristics are crossing, for example, *t* pairs, then the distribution of organisms in the progeny can be obtained from the product of *t* factors

 (8)

If the ratio of the numbers of phenotype organisms according to the dominant and recessive members of one of the *i* pairs of sings is of interest, then in the second generation the product (8) can be written in the form , where  *f* includes all the other factors of the product (8). From this expression, it follows that in the second generation the ratio of the numbers of organisms according to the dominant and recessive members of the pair *i* is a ratio of 3: 1. When splitting heterozygous organisms with formed of a third generation the ratio of the number of organisms to phenotypes  and  is 7: 3 and so on. Therefore, the ratio of the numbers of organisms in the generation *n* to the phenotypes  and in the case of poly - hybrid crossing has the same form as in the mono - hybrid crossing

 (9)

This ratio is valid for any pair of sings *i* from *t* pairs due to their independence.

If it is necessary to consider the ratio of the number of organisms to a phenotype with the simultaneous presence of several pairs of sings, then it is necessary to split all the brackets in the product (8) sequentially. For example, in the case of three pairs of sings (*t* = 3), using the notations introduced in the previous section, product (8) takes in this case the form

 (10)

It follows from equality (10) that in the case of crossing with three pairs of characteristics in the second generation, the ratio between the numbers of organisms according to the phenotypes of different possible pairs of genes has the form

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In the third generation, the product (8) is transformed so.



From the last expression, it follows that the relationship between the numbers of organisms with different genes when crossing organisms with three pairs of characters in generation 3 has the form

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In the same way, it is possible to obtain the corresponding ratios of the numbers of organisms in the crossing of organisms with *t* pairs of characters in the generation of *n*. By indexing repetitive products by the number of pairs of attributes, these relations take the form

 (11)

Just as in the case of bi – hybrid crossing, here it is possible to introduce inverse numbers of organisms according to phenotypes with different sets of genes. As can be seen from the expression (11), the number of these variables in the general case is (*t* - 1) *t* + 2. Each of the variables is proportional to  and tends to zero as the number of generations increases. The ratio of the pairs of any two variables with increasing *n* tends to unity. Therefore, in poly - hybrid crossing, with increasing *n*, the number of organisms is equalized by phenotypes with different sets of genes. Solutions for different values ​​of *m* tend in the space of the reciprocal numbers of organisms to the proper direction of the equilibrium position at the origin, which has the type of an isolated multidimensional stable node.

**CONCLUSION**

It is shown that the patterns of inheritance of features of organisms in their crossing are based on a special kind of mathematical relationships that allow us to derive algebraically the experimental patterns studied 150 years ago by Mendel. The core of these relations are geometric progressions. Using these relationships is possible to predict and investigate the inheritance processes outside the intervals of the initial data of Mendel's experiments (for any number of pairs of constant - distinguishing signs and any number of generations). The general formulas obtained are transformed continuously into the relations of Mendel in the conditions of the experiments carried out by him.

It has been established that with an increase in the number of generations in a plant population, the phenotypes of organisms in different gene sets are leveled quickly. These numbers, with an increase in the number of generations and the absence of death of organisms, synchronously increase while remaining practically equal to each other. Mathematically, this can be represented by trajectories in a multidimensional phase space with coordinates in the form of inverse numbers of phenotypes of organisms. These trajectories tend along their own direction to an isolated stable equilibrium position of the type of a multidimensional node at the origin. With the results obtained, based on G. Mendel's experiments, the Hardy - Weinberg principle is fundamentally inconsistent, constructed purely logistically without taking into account the real laws in living nature.

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I dedicate this work to Tatiana Fedorova

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